CONSTRUCTION OF FREE CASH FLOWS
A PEDAGOGICAL NOTE. PART I

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ABSTRACT

This is the first part of a paper where the construction of the free cash flow is studied. Usually a great deal of effort is devoted in typical financial textbooks to the mechanics of the calculations of time value of money equivalencies: payments, future values, present values, etc. This is necessary. However less or no effort is devoted to how to arrive at the figures required to calculate a NPV or Internal Rate of Return, IRR. In Part I, pro forma financial statements (Balance Sheet (BS), Profit and Loses Statement (P&L) and Cash Budget (CB) are presented. From the CB, the Free Cash Flow FCF, the Cash Flow to Equity CFE and the Cash Flow to Debt CFD, are derived. Emphasis is done to the reasons why some items included in the P&L and CB are no included in the FCF. Also, the FCF and the CFD are calculated with the typical approach found in the literature: from the P&L and it is specified how to construct them. In doing this, working capital is redefined: the result is that it has to include and exclude some items that are not taken into account in the traditional methods. In Part II a comparison between the proposed method to construct the above-mentioned cash flows and the ones found in the current and typical textbooks is presented.

KEYWORDS

Free cash flow, cash flow to equity, cash flow to debt, project evaluation, firm valuation, investment valuation, Net Present Value NPV assumptions.

JEL Classification: D92, E22, E31, G31

[1] This paper is based on chapter 6 of Velez-Pareja, Ignacio, *Decisiones de inversión, Una aproximación al análisis de alternativas*, CEJA, 1998. Available on line at http://www.javeriana.edu.co/decisiones/libro_on_line

INTRODUCTION

This is the first part of a paper where the construction of the free cash flow is studied. Usually a great deal of effort is devoted in typical financial textbooks to the mechanics of the calculations of time value of money equivalencies: payments, future values, present values, etc. This is necessary. However less or no effort is devoted to how to arrive at the figures required to calculate the Net Present Value NPV or Internal Rate of Return, IRR. In Part I, pro forma financial statements (Balance Sheet (BS), Profit and Loses Statement (P&L) and Cash Budget (CB) are presented. From the CB, the Free Cash Flow FCF, the Cash Flow to Equity CFE and the Cash Flow to Debt CFD, are derived. From the CB, the Free Cash Flow FCF, the Cash Flow to Equity CFE and the Cash Flow to Debt CFD, are derived. Also, the FCF and the CFD are calculated with the typical approach found in the literature: from the P&L and it is specified how to construct them. In doing this, working capital is redefined: the result is that it has to include and exclude some items that are not taken into account in the traditional methods. In Part II a comparison between the proposed method to construct the above-mentioned cash flows and the ones found in the current and typical textbooks is presented.

MODELS AND THEIR USE

Models simplify reality in order to deal with it and to understand the basics of the problem. For this reason some strong assumptions are generally stipulated. A good analyst has to know very well not only the model she constructs, but also the reality she desires to simulate. This is very important because conditions in reality have to be checked in order to define if the model complies with them or not. On the other hand, a good model should include as many elements from reality as possible, in order to confirm or predict the studied behavior. The model should recognize all the variables and their inter relationships, even if it is not possible to include or measure them in the model. The analyst should take into account all the measurable variables and should try to predict or determine the possible results from not taking into account some elements of the reality. After the model has been built, and only then, some assumptions or restrictions can be dropped out. Done this the estimation of the consequences over the behavior of the model can be studied.

This leads to the idea of two classes of models. Models for explanation and models for application.

The first class is related to the explanation of an idea or concept. In doing that, the model has to be simple, schematic and with many conditions, simplifying assumptions and restrictions. This kind of models is good to fix and define concepts.

The second class is related to the application of an explanatory model in a given reality. This kind of model has to include many of the variables and/or conditions that were disregarded in the explanatory model. Great effort has to be done to include as

many as real conditions as possible in order to make the model approximate the reality.

Many failures attributed to some traditional models are due to the wrong selection or utilization of the model. This is, an explanatory model has been used without the proper adjustment to convert it to an application model.

Many assumptions are hidden or implicit in a model. In some cases this is due to the time when the model was formulated. When computational resources were scarce or non-existent. But today, it might be unacceptable to include some implicit or explicit simplifying assumptions. The speed of the computers, the computing ability, their high storage capacity and precision allows us to include more variables and interrelationships among them at reasonable cost.

A very good example is the Net Present Value, NPV, and its formulae (including those available in any spreadsheet). This model starts from some assumptions that are not real, e. g.:

1. The discount rate is constant through the life of a project.
2. The reinvestment rate is the same as the discount rate.
3. The intermediate free cash flows are reinvested at the above-mentioned rate.
4. When dealing with mutually exclusive alternatives, the difference between the largest initial investment and the initial investment of the other alternatives is invested at the discount rate.

The purpose of this paper is to show how to take advantage of the power of spreadsheets to project pro forma financial statements, including cash budget CB and from them, deduct the free cash flow FCF of the project. In doing that, some of the implicit assumptions of the NPV will be examined.

A SHORT REVIEW

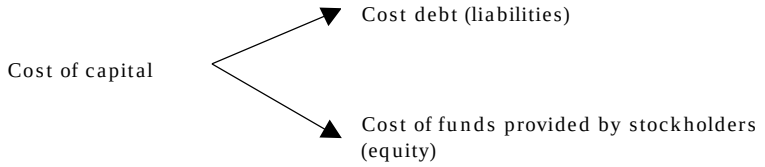
To understand the procedure to construct the FCF, it is important to review some basic concepts. First, the *discount rate*. It is an interest rate that measures the cost of money for the firm. It is the cost that the firm pays for the resources received from creditors or bondholders and from the stockholders. Usually it is known as the *Weighted Average Cost of Capital, WACC*. A second concept is the idea of *investment*: any sacrifice of resources, —money, time and assets— with the expectation to receive some benefits in the future. A third one is the distinction between the *project* (or firm), the *stockholders* and the *creditors*. Last, but not least, it is convenient to review some elementary ideas from basic accounting.

The WACC reflects the cost of debt and the opportunity cost of equity. This means that the payments for interest charges and the dividends paid to stockholders are implicit in the WACC.

Let us remember a basic concept in accounting:

$$Assets = Liabilities + Equity$$

This shows the origin of all the resources the firm has in order to make any investment. The assets the firm possesses have been acquired because there are some third parties (creditors and stockholders) that have provided for the funds to buy those assets. Every actor, creditors and stockholders, has the right to receive some return for the funds provided to the firm. Hence, the cost of capital of the firm can be visualized in this chart:



When calculating the cost of debt, interest charges and principal payments are taken into account. The cost of debt might include the cost of rent and lease as well[2]. These expenses are related to what might be called *cuasi* debt. These cost are part of the WACC. The resulting interest rate has implicit the value paid for interest charges. And these are deductible from income taxes. This means that there is a subsidy from the government for having debt and paying interest. This subsidy is equal to TxI, where T is the marginal tax rate and I is the interest payment. The net effect is a tax shield that reduces the cost of debt. A shortcut to estimate the after tax cost of debt is to calculate $i_{bt}(1\text{-}T)$, where i_{bt} is the cost of debt before taxes. This shortcut implies that taxes are paid in the same period as interest is paid.

The cost of funds provided by the stockholders might be calculated in a variety of ways. From a simple one as the well known dividend growth model up to the Capital Asset Pricing Model (CAPM). In any case, the dividends paid are taken into account in the estimation.

With these two costs (debt and equity) and the expected proportions of debt and equity, the WACC can be estimated.

[2] This idea implies that the investment should include the market value of leased or rented assets.

Let us make a very brief review of the most common financial statements utilized in the firm: The Balance Sheet and the Profit and Losses Statement (P&L) and the Cash Budget CB (not the free cash flow).

The *Balance Sheet* states and tries to measure the amount of wealth owned by stockholders. Always equilibrium is expected:

$$Assets - Liabilities = Equity$$

The *Profit and Losses Statement* tries to measure the net profit earned by the firm in a given period.

Both financial statements are constructed on the basis of the accrual concept and the assignation of costs.

This means that some income (i. e. accounts receivables) or costs (accrued payments) are registered when the situation or fact that generates the income or expense occur and not when the income is received or the expense is paid. On the other hand, cost assignation apportions some cost incurred in the past to future periods (i.e. depreciation).

The *cash budget CB* is related to the inflows and outflows of money (the checkbook movements). It measures the liquidity of the firm for each period. Usually, this is a projected or pro forma financial statement. In this financial statement all the expected inflows and outflows are registered when they are expected to occur. It is better to analyze the liquidity situation of the firm with this pro forma statement rather than with the traditional financial ratios. Usually what is known as financial analysis is a kind of autopsy of the firm. They look at the past, assuming that the past will be repeated in the future. Cash budget CB is one of the most, if not the most important tool to control and follow up the liquidity of the firm.

For investment analysis or project evaluation, it is important to know how is the performance of the firm or project in terms of liquidity. A careful examination of the cash budget CB will allow the decision maker to choose a given financing alternative or, on the other hand, a good investment of cash surplus decision.

The elements of the cash budget CB are the inflows and the outflows of cash. The difference between these two figures result in the cash balance of the period and from it, the cumulative cash balance can be calculated. This cash balance is the clue to decide if the firm should borrow or invest funds.

Typical items included in a pro forma cash budget CB are:

Inflows	Outflows
Accounts receivables recovery	Accounts payable payments
Loans received	Salaries and fringe benefits
Equity invested	Interest charges
Sale of assets	Principal payments
Return on short or long term investment	Rent
Short term investment recovery	Overhead expenses
Customers' in advance payments	Promotion and publicity
Repayments of cash lent	Asset acquisition
	Social Security payments
	Earnings distributed or dividends paid
	Taxes
	Short term investment of cash surpluses

And last, but not least, the *free cash flow (FCF)*. The FCF measures the expected operating benefits and costs of an activity (firm or project). In contrast with the cash budget CB, some cash inflows or outflows are excluded from the FCF. More, some items included in the FCF of a project might not be a real cash inflow or inflow (i.e. the opportunity cost of some resource: remember the definition of investment: a sacrifice of resources). With the FCF, the NPV and IRR are calculated.

Along with the FCF two other cash flows are constructed: the financing cash flow or cash flow to debt CFD and the owner or stockholder cash flow or cash flow to equity CFE.

WHAT IS INCLUDED IN THE FCF

In the cash budget CB are included all the inflows and outflows. Among them are found some items such as, equity investment, proceeds from loans, principal payments, interest charges, earnings distributed or dividends and implicit in the tax payments, the tax shield for interest payments.

These above-mentioned items should not be included in the FCF, as follows.

1. Equity payment
2. Loans received
3. Loans paid

These are not included in the FCF because they are not the result of the operating activity of the firm or the project. For instance, the inflow of a loan is not a benefit produced by the firm in its operations. The idea is to measure the value generated by the firm or project.

4. Interest charges paid
5. Tax shield for interest payments
6. Distributed earnings or dividends

These items are not included in the FCF because they are embodied in the cost of capital. Including these items would result in a double counting of the cost of the money for the firm or project. The FCF must be discounted with the WACC in order to calculate the NPV and the IRR is compared to the WACC in order to know if the project is accepted or rejected.

The FCF is called free because it must be *clean* from any financing effect, including the effect from the tax payment (the tax shield). The financing effects are embodied in WACC.[3]

INTEREST PAYMENTS AND EARNINGS PAID

When applying the discounted cash flow analysis (DCF) the idea behind it is to determine if there exists value added. This value added comes from the operating net benefits and any other operations performed by the firm. In order to accept a project it has to be able to pay back the amount invested plus the cost of money (the discount rate or WACC). (See Velez-Pareja, 1999) The discounting process ($P=F/(1+i)^n$) discounts the interest paid at the discount rate. As it was mentioned above, the WACC is calculated taking into account the interest and earnings or dividends paid. This means that if the cost of debt or *cuasi* debt (interest, rent and leases charges) and part of the cost of equity (earnings or dividends paid) are included in the FCF as an outflow and besides the NPV is calculated ($P=F/(1+i)^n$), the cost of money would be counted twice. Hence, there would be an underestimation of the value added (NPV) by the alternative analyzed. For the same reason, the tax shield has to be excluded from the FCF: it has been measured in the cost of debt after taxes and is implicit in the WACC.

A simple example will clarify the idea.

EXAMPLE 1

Assume an investment of $1,000 and $1,500 are received in a year.

0	-1.000
1	1.500

If the WACC is 30%, then the $1,500, might be decomposed as follows:

	1,000
	300
	200

When discounted, the present value of $1,500 is $1,153.85. This figure can be decomposed as,

	Cash flow in year 1 $	Present value $	Cumulated value
Investment $	1,000	769.23	769.23
Cost of money $	300	230.77	1,000
Value added $	200	153.85	1,153.85

Investment plus the cost of money (the interest charged to the project) are equivalent to $1,000 in year zero. This means that the discounting process discounted (subtracted) the interest charged for the investment (the cost of money or WACC. The

[3] There is a debate upon the suitability of WACC to evaluate an investment alternative. Alternate methods are the Adjusted Present Value APV (Myers, 1974) and the Generalized Adjusted Present Value GAPV (Gallo and Pecatti, 1993 and Prina della Tallia and Pattison, 1996)

remaining, when discounted is precisely the NPV. And this value —$153.85— at year zero means that it is a good project and should be accepted.

On the other hand if the interest (the $300) is subtracted in the FCF, the cash flow would be.

	FCF $	Interest payments $	Net FCF $
0	-1,000		-1.000
1	1,500	-300	1.200

When discounted at 30%, the NPV will be -76.92 and the project has to be rejected.

Again, if the interest payments are subtracted in the FCF and the NPV is calculated, the cost of money will be counted twice. And with this approach we could reject a good project.

LOANS AND EQUITY

If the loans are included as an inflow and later as an outflow, the resulting cash flow is not the free cash flow of the project. The investment in a project is the total value of all the resources sacrificed in it, no matter where they come from. On the other hand, loan or equity inflows are not part of the free cash flow of the project because they are not a benefit produced by the operation of the project. By the same token, the loan payments are not an expense of the operation of the project. It is necessary to recall that the idea is to evaluate how good is the project. How much value it aggregates from the operation to the firm.

TAXES

All taxes have to be included in the free cash flow. This means local taxes, capital gains taxes, income tax, etc. The best way to determine the taxes paid by the project is to calculate the taxes of the firm with and without the project. The taxes attributable to the project are the difference between the taxes paid by the firm with the project and the taxes paid by the firm without the project.

In countries where adjustments for inflation are applied to the financial statements, the only adjustment to be done in the FCF is the one related to taxes paid after adjustments for inflation. It has to be remembered that adjustments for inflation do not create wealth, they only try to approximate to reality.

The net effect of taxes on the firm is that an expense after taxes is equal to the same expense minus the tax shield or tax savings. A deductible expense (i.e. interest payments) implies tax shield or tax savings of T times I, where T is the marginal tax rate and I is the interest charges paid.

CONSTRUCTION OF THE FREE CASH FLOW FCF

In the case of the discounted free cash flow approach (DCF) for project evaluation there are many oversimplifying assumptions or shortcuts. These shortcuts were valid 35 or 100 years ago. However, they can be found today in finance and engineering economy textbooks, and among teachers, analysts, managers and practitioners. For the projection of pro forma financial statements a great variety of "methods" exist. The typical methodology found in many respected textbooks is to express the items of the last balance sheet and P&L statement as a percentage of net sales, make a lineal regression for sales with a few data and apply the percentages to the projected sales figures. How to balance the figures? Easy, they say, use cash and current liabilities or dividends as the checking or balancing account![4]

When a project is planned (to start a business, for instance) it is necessary to construct the initial (year 0) Balance Sheet and gather some information about the market, sales volume, price-demand elasticity, unit sales price, unit input price, increase in the level of sales, price increases, etc. With this information pro forma financial statements are projected. From the P&L statements and the plans and policies for collecting receivables and paying accounts payable, inventories, dividends or earnings payment, etc. the cash budget (CB) can be constructed. These are managerial tools for follow up and control a project or firm.

The CB figures are the closest to the FCF figures of a project because the CB registers the inflows and outflows at the time they occurs. Hence, it is from this financial statement where the FCF should be derived.

THE FREE CASH FLOW (FCF) OF A PROJECT

In order to accept or reject an alternative, it is necessary to measure if the projected cash flows are enough to generate aggregated value to the firm. This is done with the free cash flow (FCF) of the project. The FCF can be deducted from the pro forma financial statements.

As was studied above, it is necessary to eliminate from the CB some items that should not be included in the FCF. These items are loans received and paid, equity invested by stockholders, dividends paid to stockholders, interest charges paid and the tax shield from interest payments. Then, for year 1 to n:

Net cash gain (loss) after financing and reinvestment from the CB (NCG)
Minus investment from stockholders (IS)
Minus proceeds from loans received (LR)
Plus principal payments (PP)
Plus interest, rent and lease charges paid (I)
Minus tax shield for interest. Rent and lease payments (TSI)
Plus dividends or earnings paid (D)
Minus investment in the project (IP)
Equal free cash flow

[4] Van Horne 1998, p. 742, Gallaher and Andrew 2000, p. 129 and Brealey, Myers and Marcus, 1995, p. 521 in the Spanish version

At period n, the present value of future FCF beyond the period n, has to be added. Copeland et.al. (1995) and Weston and Copeland (1992), call it continuing value. Damodaran (1996) calls it terminal value[5]. For year 0 the opportunity cost of the resources engaged in the project. In other words, total assets. With the FCF the NPV is calculated at the weighted average cost of capital WACC. It should be remembered that the WACC includes the cost of debt and the cost of equity.

EXAMPLE 2

Assume a new commercial business is planned. Based on forecasts, the financial statements are projected, as follows[6]:

Table 1

Pro forma Balance sheet	Year 0	Year 1	Year 2	Year 3	Year 4
Assets					
Cash	110.0	110.0	121.0	150.0	65,758.9
Accounts receivable	-	2,525.9	3,358.3	4,311.0	5,408.4
Inventory	-	2,052.3	2,735.3	3,370.6	4,140.1
Investment	-	-	8,486.0	32,969.9	-
Interest accrued	-	-	-	-	-
Fixed assets	40,000.0	40,000.0	40,000.0	40,000.0	40,000.0
Cumulative depreciation	-	8,000.0	16,000.0	24,000.0	32,000.0
Net fixed assets	40,000.0	32,000.0	24,000.0	16,000.0	8,000.0
Total assets	40,110.0	36,688.2	38,700.7	56,801.6	83,307.4
Liabilities and equity					
Accounts payable suppliers	-	2.668,0	3.298,8	4.058,9	4.982,8
Accounts payable overhead		214.0	265.4	326.4	401.5
Accounts payable fringe benefits		376,3	475,4	587,6	720,6
Accrued taxes	-	505.7	3,731.3	8,582.0	14,342.6
Long term debt	16,110.0	8,081.2	121.0	-	-
Total liabilities	16,110.0	11,845.3	7,891.8	13,554.9	20,447.5
Equity	24,000.0	24,000.0	24,000.0	24,000.0	24,000.0
Retained earnings	-	-	590.0	4,943.2	14,955.6
Earnings for the period	-	842.9	6,218.8	14,303.4	23,904.3
Total	40,110.0	36,688.2	38,700.7	56,801.6	83,307.4

[5] In these three references there is a complete and comprehensive study of this issue.
[6] A detailed version including the parameters to reach these figures is available from the author on request.

Table 2

Pro forma Profit and Losses Statement	Year 0	Year 1	Year 2	Year 3	Year 4
Sales		50,518.1	67,165.8	86,219.9	108,168.2
Cost of goods sold		24,628.1	32,304.7	39,953.3	49,058.6
Gross profit		25,890.0	34,861.1	46,266.6	59,109.6
Selling and administrative expenses		19,297.1	22,304.5	25,743.0	29,830.5
Depreciation		8,000.0	8,000.0	8,000.0	8,000.0
Earnings before interest and taxes		6,592.9	12,556.6	20,523.6	29,279.1
Other income (interest received)		-	-	2,398.2	8,967.8
Other expenses (interest expenses)		5,244.2	2,606.6	36.3	-
Earnings before taxes		1,348.7	9,950.1	22,885.5	38,246.9
Taxes		505.7	3,731.3	8,582.0	14,342.6
Net profit		842.9	6,218.8	14,303.4	23,904.3

Table 3

Pro forma cash budget CB					
	0	1	2	3	4
Cash balance at start of year	-	110.00	110.00	121.00	150.00
Cash inflows					
Cash collection of sales		47,992.2	66,333.4	85,267.2	107,070.8
Recovery of short term investments[7]	-	-	-	8,486.0	32,969.9
Interest on short term investment	-	-	-	2,398.2	8,967.8
Equity investment	24,000.0	-	-	-	-
Total cash inflows	24,000.0	47,992.2	66,333.4	96,151.4	149,008.5
Cash outflows					
Payments to suppliers	-	24,012.4	32,357.0	39,828.5	48,904.1
Administrative and sales expenses	-	10,706.8	14,154.0	17,569.8	21,622.4
Purchase of fixed assets	40,000.0	-	-	-	-
Interest charges	-	5,244.2	2,606.6	36.3	-
Dividend payments	-	-	252.9	1,865.6	4,291.0
Taxes	-	-	505.7	3,731.3	8,582.0
Total cash outflows	40,000.0	39,963.4	49,876.2	63,031.5	83,399.6
Net cash gain (loss)	-16,000.0	8,028.8	16,457.3	33,119.9	65,608.9
Cash balance at end of year	-16,000.0	8,138.8	16,567.3	33,240.9	65,758.9
Proceeds from loans	16,110.0	-	-	-	-
Principal payments	-	8,028.8	7,960.2	121.0	-
Investment of surplus	-	-	8,486.0	32,969.9	-
Net cash gain (loss) after financing and reinvestment	110.0	-	11.0	29.0	65,608.9
Cumulative cash balance at end of year after financing and reinvestment	110.0	110.0	121.0	150.0	65,758.9

Notice that taxes are paid the following year after accrued. This is important because the tax shield is received only at the time taxes are paid.

Now, the free cash flow is estimated. With the rates of discount for each year, the NPV is calculated.

[7] Rigorously, the cash flows associated to the investment of cash surplus and its returns must be discounted at a different discount rate. For a complete discussion of this issue see Copeland et.al. (1995) and Weston and Copeland (1992).

Table 4

Free cash flow for the firm $						
For year 1 to n						
Net cash gain (loss) after financing and reinvestment	(Plus)	-	-	11.0	29.0	65,608.9
Proceeds from loans LR	(Minus)	-	-	-	-	-
Principal payments PP	(Plus)	-	8,028.8	7,960.2	121.0	-
Equity investment	(Minus)					
Dividends	(Plus)	-	-	252.9	1,856.1	4,291.0
Interest charges	(Plus)	-	5,244.2	2,606.6	36.3	-
Tax shield for interest payments	(Minus)		-	1,966.6	977.5	13.6
For period n, terminal or market value (growth at n+1 = 30%, afterwards 0%)	(Plus)					82,752.5
For year 0: total assets						
Free cash flow after taxes		-40,110.0	13,273.0	8,864.1	1,074.5	152,638.8
Discount rate after taxes (max(WACC, short term rate)) %			38.97%	38.76%	34.18%	32.78%
Present value discount factor		1.0	0.7196	0.5186	0.3865	0.2911
NPV		18,883.7				

The project should be accepted.

THE CASH FLOW TO EQUITY (CFE)

When evaluating investment alternatives that imply financing by debt, it is possible to interpret erroneously the benefits of the project. This risk is present when cash flows are mixed up. This is, when the CFD is mixed with or included in the FCF. Remember that FCF = CFD + CFE. If CFD is included in FCF, then the result is just the CFE. If this cash flow is accepted as the FCF, then the analysis is not for the project, but for the funds invested by the stockholders. It is very important to evaluate the CFE, but the analysis of the CFE as if it were the FCF, might produce misleading results. To make IRR calculations with the CFE, assuming that it is the FCF, overvalue the rate of return and the NPV for the project. This overvaluation of the desirability of a project evaluated with wrong cash flow is due to the leverage. A simple example will illustrate the idea.

EXAMPLE 3

Assume that a firm sells and pays on a cash only basis. The stockholders invested $1,000,000. The cost of equity is 50%. Assume no taxes. The resulting cash flows are:

Inflows	2,500,000
Outflows	900,000
Net income	1,600,000

The IRR for the firm is 60% and it is the same for the stockholders. If the operation is financed 50% debt, 50% equity and the cost of debt is 30%, then,

Inflows $	2,500,000
Outflows $	900,000
Principal payment $	500,000
Interest $	150,000
Cash balance $	950,000

The FCF is the same as before and the IRR is, again, 60%. However, when the stockholder equity is $500,000, the IRR for her is greater than 60% and in this example is 90%.

	No debt	Debt 50%
IRR for the firm	60%	60%
IRR for stockholders	60%	90%
IRR for debt		30%
Cost of equity	50%	70%
Cost of debt	30%	30%
WACC	50%	50%
NPV for project[8]	66,67	66,67
NPV for stockholder	66,67	58,82

As can be seen if the cash flows are mixed up, it is possible to calculate an IRR for the project equals to infinity (when the equity is 0).

From the stockholders point of view, the cash flow associated to them are the equity paid by them, the end of period cash surpluses after financing or investing, the dividends or earnings paid and finally, the end cash balance at period n.

The cash flow to equity ¾ CFE¾ equals to net cash gain (loss) after financing and reinvestment minus their investments plus dividends or earnings paid ¾ for periods 0 to n-1¾ . For period n is cumulative cash balance at end of year after financing and reinvestment minus equity invested plus dividends or earnings paid.

In summary, for periods 1 to n-1

Net cash gain (loss) after financing and reinvestment from CB (NCG)
Minus investment from stockholders (IS)
Plus dividends or earnings paid (D)

For period 0, the total investment from stockholders.

For period n:

Cumulative cash balance at end of year after financing and reinvestment (CCB)
Minus investment from stockholders (IS)
Plus net terminal value (NSV)
Plus dividends or earnings paid (D)

In the example:

[8] In a Modigliani-Miller world the WACC is unchanged and this means that the NPV for the firm is unchanged as well.

Table 5

Cash flow to equity CFE $		Year 0	Year 1	Year 2	Year 3	Year 4
For year 0 to n-1						
Net cash gain (loss) after financing and reinvestment	(Plus)	0.0	0.0	11.0	29.00	65,608.9
Dividends	(Plus)		0.0	252.9	1,856.1	4,291.0
Equity investment	(Minus)	24,000.0	0.0	0.0	0.0	
Terminal or market value or continuing value	(Plus)					82,752.5
Equity cash flow after taxes		-24,000.0	0.00	263.87	1,894.64	152,652.4

If equity is $40.000 and there is no debt, then the NPV of the project and the NPV to the stockholders are equal, because the two FCF are identical and the WACC is the same as the cost of equity (no debt).

THE CASH FLOW TO DEBT, CFD

This cash flow is simply the inflows for debt borrowed and the outflows for interest and principal payments. The tax shield from interest payments has to be included in the cash flow (this is necessary if the analysis is after taxes).

In the example:

Table 6

	Year 0	Year 1	Year 2	Year 3	Year 4
Cash flow to debt CFD $	-16,110.0	13,273.0	8,600.2	-820.1	-13.6

RELATIONSHIP BETWEEN CASH FLOWS

When analyzing a project three "actors" are found: the project or firm, creditors (bondholders, banks, etc.) and stockholders. For each one there is a cash flow. For the project the associated cash flow is the free cash flow (FCF), free from financing effects. With this cash flow the firm measures the added value produced by the firm or project. This increase will be reflected in the financial statements in the long run. In fact, the value added (NPV) is finally obtained when the project is finished. The cash flow to the stockholders (or cash flow to equity, CFE) reflects the amounts invested and the dividends or earnings received by them. Finally, the financing cash flow (or cash flow to debt, CFD) which includes the tax savings for payment of interest (tax shield).

From basic accounting this equation holds:

$$Assets = Liabilities + Equity$$

For each of the elements of this equations there is a cash flow associated. The assets are what is invested in the firm or project, hence, the associated cash flow is the free cash flow (FCF). For liabilities, the associated cash flow is the cash flow to debt (CFD) and for equity, it is the cash flow to equity (CFE). The same relationship holds between cash flows:

$$\text{Free cash flow} = \text{Cash flow to debt} + \text{cash flow to equity}$$

$$FCF = CFD + CFE$$

When the *Weighted Average Cost of Capital WACC* was studied above, the same basic relationship held. It is a mix (of debt and equity). Hence, the discount rate to discount FCF is

the WACC. For the FCD the discount rate is the after tax cost of debt and for the CFE the discount rate is the cost of equity.

In the example:

Table 7

	Year 0	Year 1	Year 2	Year 3	Year 4
FCF $	-40,110.0	13,273.0	8,864.1	1,074.5	152,638.8
CFD $	-16,110.00	13,273.0	8,600.2	-820.1	-13.6
CFE $	-24,000.00	0.00	263.9	1,894.6	152,652.4
CFD + CFE $	-40,110.00	13,273.0	8,864.1	1,074.5	152,638.8

This approach has some advantages:

It provides managerial tools. The construction of the different financial statements provides managerial tools for control and follow up.

It allows for immediate checking for consistency. The Balance Sheet and the relationship between cash flows provide a checking point to guarantee that the different cash flows and pro forma financial statements are correct.

It is simple. The adjustments made refer to "real" figures found at the cash budget CB, except a "virtual" adjustment to the taxes paid (the tax shield).

It is a better approach to what happens in reality. In the model, different real assumptions can be introduced, i.e. the reinvestment of cash surpluses.

It provides a tool for sensitivity analysis and scenario analysis. The spreadsheet where these figures come from includes all the relationships between basic variables such as, prices, increases y prices, volume, increase in volume, price-demand elasticity, accounts receivable, accounts payable and inventory policies, cash cushion policy, etc.

CALCULATION FROM THE P&L STATEMENT

The ***free cash flow*** -FCF- can be deducted from the Profit and Losses Statement (P&L). The right arithmetic for it is, starting from EBIT (for periods 0 to n-1):

For year 0: total assets.

For years 1 to n:

Earnings before interest and taxes (EBIT)
minus taxes on EBIT (TEBIT)[9]
plus depreciation charges (DC)
minus change in working capital[10] *(CWC)*
plus returns on short term investment x (1-Tax rate) (NRI)
minus investment in the project (IP)

For year n, add *net salvage (or market value or continuing value) (NSV).*

In the example:

Table 8

	Year 1	Year 2	Year 3	Year 4
Accounts receivable + inventories + short term investment of cash surplus	4,578.2	14,579.7	40,651.6	9,548.5
Accounts payable (non-interest bearing interest)	3,764.1	7,770.8	13,554.9	20,447.5
Tax shield from interest payments	1,966.6	977.5	13.6	0.0
Working capital	-1,152.4	5,831.4	27,083.0	-10,899.0
Change in working capital CWC	-1,152.4	6,983.8	21,251.6	-37,982.0
EBIT	6,592.9	12,556.6	20,523.6	29,279.1
Tax rate T	37.5%	37.5%	37.5%	37.5%
Tax on EBIT (T x EBIT)	2,472.3	4,708.7	7,696.4	10,979.7
Interest expenses I	5,244.2	2,606.6	36.3	0.00
Depreciation DC	8,000.0	8,000.0	8,000.0	8,000.0
Net return on short term investment RIX(1-T)			1,498.8	5,604.9
Terminal or market value or continuing value *NSV*				82,752.5
FCF = EBIT –T x EBIT + DC – CWC + RI x (1-T) + (for year 4) NSV	13,273.0	8,864.1	1,074.5	152,638.8

If calculated from net profits, (for periods 1 to n):

Net profit (NP)
plus depreciation charges (DC)
plus interest charges x (1-tax rate) (NIC)
minus change in working capital[11] *(CWC)*
minus investment in the project (IP)

For year n, add *net salvage (or market value or continuing value)(NSV).*

In the example:

[9] When working from EBIT care has to be taken in order not to include taxes on EBIT if there are no taxes to pay.

[10] Working capital is defined as receivables plus inventories plus short-term investments minus accounts payable minus tax shield for interest payments. (Cash is not included). The assumption is that taxes are paid the year after accrued.

[11] Working capital is defined as receivables plus inventories plus short term investments minus accounts payable minus tax shield for interest payments. (Cash is not included). The assumption is that taxes are paid the year after accrued.

Table 9

	Year 1	Year 2	Year 3	Year 4
Net profit NP	842.9	6,218.8	14,303.4	23,904.3
Depreciation DC	8,000.0	8,000.0	8,000.0	8,000.0
Change in working capital CWC	-1,152.4	6,983.8	21,251.6	-37,982.0
Interest expenses I	5,244.2	2,606.6	36.3	0.00
Tax shield from interest payments	1,966.6	977.5	13.6	0.0
FCF = NP +DC –CWC + Ix(1-T)	13,273.0	8,864.1	1,074.5	152,638.8

In both cases, ***working capital is defined as receivables plus inventories plus short term investments minus accounts payable minus tax shield for interest payments. (Cash is not included)***. Cash is not included in the current assets because calculation of the working capital and its change from period to period is done in order to take into account the funds accrued, but not received or paid in the period. As cash is already in the firm (in the bank or anywhere), it is not necessary to be included in the calculation of future funds. On the other hand, the reason why the tax shield has to be included as part of the working capital is that in both approaches – from EBIT or from net profits– the taxes are calculated or adjusted to the taxes on EBIT. As the taxes included in the current assets are the taxes calculated on earnings after interest charges, the tax shield has to be recovered. Also, care has to be taken in verifying that taxes on Operating Profits or Net Profit before taxes really occurs. It depends on the local tax law. It should be remembered that in some places it is not possible to have a carry over, in case of losses.

The ***cash flow to equity*** CFE can be deducted from the P&L statement, as follows:

For year 0, the stockholders investment

For years 1 to n from EBIT:

Earnings before interest and taxes EBIT
minus taxes on EBIT TEBIT
plus depreciation charges DC
minus change in working capital[12] *(CWC)*
plus returns on short term investment x (1-Tax rate) NRI
plus proceeds from new debt DT
minus principal payments PP
minus interest charges I
Plus tax shield for interest paid previous year (n-1) IxT[13]
minus investment in the project IP

For year n, add *terminal value (or market value or continuing value) NSV.*

From net profits:

Net profit NP
plus depreciation and amortization DC
plus proceeds from new debt DT
minus principal payments PP

[12] Working capital is defined as receivables plus inventories plus short term investments minus accounts payable minus tax shield for interest payments. (Cash is not included).

[13] The assumption is that taxes are paid the year after accrued.

minus change in working capital CWC
minus investment in the project IP

In the example

Table 10

	Year 1	Year 2	Year 3	Year 4
Accounts receivable + inventories + short term investment of cash surplus	4,578.2	14,579.7	40,651.6	9,548.5
Accounts payable (non-interest bearing interest)	3,764.1	7,770.8	13,554.9	20,447.5
Tax shield from interest payments	1,966.6	977.5	13.6	0.0
Working capital	-1,152.4	5,831.4	27,083.0	-10,899.0
Change in working capital CWC	-1,152.4	6,983.8	21,251.6	-37,982.0
Proceeds from new debt LR	0	0	0	0
Principal payments PP	8,028.77	7,960.23	121.00	0.00
Interest expenses I	5,244.2	2,606.6	36.3	0.00
Depreciation DC	8,000.0	8,000.0	8,000.0	8,000.0
Net return on short term investment RIX(1-T)			1,498.8	5,604.9
Terminal or market value or continuing value *NSV*				82,752.5
Investment by stockholders IS	0	0	0	0
FCE = NP + DC – WCW – PP + LR - IS	0.0	263.9	1,894.6	152,652.4
FCE = EBIT(1-T) + DC – WCW – PP + LR – IS + RI(1-T)-I + TXI(n-1) +NSV	0.0	263.9	1,894.6	152,652.4

Working capital is defined as ***receivables plus inventories plus short term investments minus accounts payable minus tax shield for interest payments. (Cash is not included)***.

For checking purposes:

Table 11

Cash flows derived from P&L statement				
	Year 1	Year 2	Year 3	Year 4
FCF	13,273.0	8,864.1	1,074.5	152,638.8
CFE	0.0	263.9	1,894.6	152,652.4
CFD	13,273.0	8,600.2	-820.1	-13.6
CFD +CFE	13,273.0	8,864.1	1,074.5	152,638.8

Notice that the FCF and the FCE calculated from P&L statement are the same as the ones calculated from cash budget. These last approaches from P&L are conceptually right, but they have to be carefully done, trying to include any credit received or given to customers and any other item as mentioned above. However, they disregards possibility to construct and use the cash budget CB the project or firm. This is a very useful tool for the management and should be used as much as possible.

Many teachers, analysts and practitioners do not include the change in WC. Others subtract the principal and interest payments. More, some of them do not distinguish

between the cash budget CB and the free cash flow FCF. Others, (i.e. Blank and Tarquin[14], 1998) do not make a difference between the free cash flow and the cash flow to equity. Some others define the working capital as current assets (including cash) minus current liabilities.

THE NPV REINVESTMENT ASSUMPTION

The NPV assumes that the free cash flows FCF are reinvested at the discount rate. Let us analyze this assertion.

- First, the model assumes correctly that any treasurer does not freeze the cash in the bank or safe. True. This is correct.
- Second, the model assumes that the free cash flow FCF is reinvested. False. As can be seen in the example (and in reality), the reinvested cash flows are not from the free cash flows FCF, but from the cash budget CB, which might be quite different.
- Third, the model assumes that the reinvestment rate is the discount rate. False. In general, the reinvestment rate is not the discount rate. Usually, the rate for reinvestment is the short term rate. It can be named the opportunity rate for the firm. However, it might be possible to find a reinvestment rate even greater than the discount rate.

The model presented for constructing the free cash flow FCF, allows for the forecasting of reinvestment rates (greater than, equal to or less than the discount rate). However, in general, if the different rates are ranked (according to the level of risk and assuming that the reinvestment rate is less than the discount rate), the order would be as follows:

$$i_e > WACC > i_d > i_r > i_f$$

where i_e is the opportunity cost of equity, WACC is the weighted average cost of capital, i_d is the cost of debt, i_r is the reinvestment rate and i_f is the risk-free rate of interest.

If the BC and the FCF are examined, it will be found that the difference between the figures accounts for the dividends, the interest paid, taxes (and the tax shield as less taxes) and the "cushion" of security cash balance. These amounts are not available for the firm. They have been paid out or are "frozen" at the bank. It is not reasonable to assume that they can be reinvested at the discount rate. If these amounts are left as free cash flows FCF, there would be returns that will not occur. Hence, it is necessary to rearrange the FCF in order to recognize that fact. This can be done "reinvesting" intermediate cash flows at 0% and recalculating the NPV at the discount rate(s).

In the example:

Table 12

	Year 0	Year 1	Year 2	Year 3	Year 4
Modified FCF with cash flows reinvested at 0%	-40,110.0	0.0	0.0	0.0	175,850.4

[14] This book is of great acceptance in the academic and practitioners world in Colombia.

While the NPV from the original FCF was 18,883.7, now, and from the modified FCF, the NPV is 11,177.2, lower that the former, as expected.

This means that in order to calculate the "real" NPV, the reinvestment assumption has to be carefully examined.

SUMMARY

A method to calculate the FCF from the CB is presented. Methods to calculate the FCF from the P&L statement are presented as well. These methods leads to the same FCF if careful done. The reinvestment assumption, implicit in the NPV calculation is examined.

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